MTH441 Exam 2 Things

1. Suppose R is a rectangle and f, g : R → **R** are integrable. Show that the function

h(x) = max{f(x), g(x)} is integrable on R.

[If you don’t use partitions, you’re doing it wrong.]

Let ep>0

Since f and g are integrable on R, there are paritions N1 N2 so

S(N1,f) – s(N1,f) < ep and S(N2,g) – s(N2,g) < ep

Let N be the common refinement of N1, N2

Let Mij = max(sup(f on Rij), sup(g on Rij))

∑(Mij \* A(Rij)) - ∑(mij \* A(Rij)) < ep

2a.[Related to 2b.] Suppose that f : Rn → [0, ∞) is continuous and satisﬁes lim |p|→∞ f(p) = 0

Prove that f has a maximum value on Rn. That is prove that there is a p ∈ Rn such that

f(p) ≥ f(x) for all x ∈ Rn.

Let ε>0. There is a R so for all |p|>R so f(p) < ε

Consider the closed ball B(0, R).

2b.[Related to 2a] Give an example of a continuous function f : R → R such that lim

|x|→∞

f(x) = 0

but f has no maximum value on R.